Monday, August 31, 2015

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Problem 3

Problem. Evaluate $\ln 45$ using (a) a calculator and (b) numerical integration.

Solution. (a) The TI-83 reports that $\ln 0.8 = -0.2231435513$.

(b) Let `Y1=1/X`. Graph the function and then integrate it from 1 to 0.8 (note the reverse order). The TI-83 reports a value of -0.2231436.

Problem 5

Problem. Match the function $f(x) = \ln x + 1$ with its graph.

Solution. The function matches (b). It is the basic graph of $y = \ln x$ raised up one unit.

Problem 7

Problem.

Match the function $f(x) = \ln(x - 1)$ with its graph.

Solution. The function matches (a). It is the basic graph of $y = \ln x$ shifted right one unit.

Problem 9

Problem. Sketch the graph of $f(x) = 3 \ln x$ and state its domain.

Solution. The graph of $f(x) = 3 \ln x$ is the basic graph of $y = \ln x$ stretched vertically by a factor of 3. Its domain is the same as the domain of $\ln x$, namely, $(0, \infty)$.

Problem 12

Problem. Sketch the graph of $f(x) = \ln |x|$ and state its domain.

Solution. The graph is in two parts. To the right of the y-axis, the graph is the same as the graph of $y = \ln x$. To the left of the y-axis, the graph is the mirror image of the graph on the right. The domain is all real numbers except 0.

Problem 18

Problem. Use the properties of logarithms to approximate the following logarithms, given that $\ln 2 \approx 0.6931$ and $\ln 3 \approx 1.0986$.

(a) $\ln 0.25$

(b) $\ln 24$

(c) $\ln \sqrt[3]{12}$

(d) $\ln \frac{1}{72}$

Solution. (a) $\ln 0.25 = \ln \frac{1}{4} = -\ln 4 = -\ln 2^2 = -2\ln 2 \approx -2(0.6931) = -1.3862$.

(b) $\ln 24 = \ln (2^3 \cdot 3) = 3\ln 2 + \ln 3 \approx 3(0.6931) + (1.0986) = 3.1779$.

(c) $\ln \sqrt[3]{12} = \ln 12^{1/3} = \frac{1}{3}\ln (2^2 \cdot 3) = \frac{1}{3}(2\ln 2 + \ln 3) \approx \frac{1}{3}(2(0.6931) + 1.0986) = 0.8283$.

(d) $\ln \frac{1}{72} = -\ln 72 = \ln (2^3 \cdot 3^2) = -(3\ln 2 + 2\ln 3) = -(3(0.6931) + 2(1.0986)) = -4.2765$.

Problem 21

Problem. Use the properties of logarithms to expand the logarithmic expression $\ln \frac{xy}{z}$.

Solution.

$$\begin{aligned} \ln \frac{xy}{z} &= \ln xy - \ln z \\ &= \ln x + \ln y - \ln z. \end{aligned}$$

Problem 26

Problem. Use the properties of logarithms to expand the logarithmic expression $\ln 3e^2$.

Solution.

$$\begin{aligned} \ln 3e^2 &= \ln 3 + 2\ln e \\ &= \ln 3 + 2(1) \\ &= 2 + \ln 3. \end{aligned}$$

Problem 29

Problem. Use the properties of logarithms to write

$$\ln(x-2) - \ln(x+2)$$

as a logarithm of a single quantity.

Solution.

$$\ln(x-2) - \ln(x+2) = \ln\frac{x-2}{x+2}.$$

Problem 31

Problem. Use the properties of logarithms to write

$$\frac{1}{2}[2\ln(x+3) + \ln x - \ln(x^2-1)]$$

as a logarithm of a single quantity.

Solution.

$$\begin{aligned}\frac{1}{2}[2\ln(x+3) + \ln x - \ln(x^2-1)] &= \frac{1}{2}\ln\frac{(x+3)x}{x^2-1} \\ &= \ln\sqrt{\frac{(x+3)x}{x^2-1}}.\end{aligned}$$